OPTIMIZATION OF CRITICAL SPEEDS IN A ROTOR DYNAMIC SYSTEM BY APPLYING THE MULTI-OBJECTIVE GENETIC ALGORITHMS

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ABSTRACT At high rotational speeds, the rotor dynamic systems exhibit complex behaviors which may lead to severe dangerous and unrecoverable damages. While designing rotors, it is extremely important to consider the rotor dynamic characteristics into account for safe operation. However, determining these characteristics is difficult as the systems in general have complex geometries. To get better products, the better analysis tools are always required to optimize and to make them more stable. Recently, it is possible to obtain faster solutions by using software that can solve the complicated governing equations of the systems. The main goal of this paper is to propose a method to investigate and optimize a rotor dynamic system in stability at the operating speeds. For this purpose, the Campbell diagram is utilized to find critical frequencies of the system by the ANSYS finite element program. The modal analysis of the system was performed by finite element analysis (FEA) using ANSYS parametric design language (APDL). In particular, the 3D rotor finite elements were modeled as Timoshenko beams including the effects of rotary inertia, unbalance force, gyroscopic effects. The idea of the optimization problem is to find the optimal set variables (e.g. stiffness of the bearing, cross sections of the rotor, etc.) for which the critical frequencies are as far as possible from the operating speeds of the machine. Therefore, the design of experiments (DOE) and response surface methods (RSM) were carried out to provide the response points. As to the design procedures, multi-objective genetic algorithms (GAs), a robust and stochastic algorithm based on the natural selection, were applied to find the optimum variables of the system. The results show that the critical frequencies of the system can be significantly changed by slight modification of the variables.

1. INTRODUCTION

Recently, the study of rotor dynamic systems plays an extremely significant role in some industrial fields, such as aerospace, electronics, printed circuit boards (PCB). Devices should be analyzed characteristics to improve the design and reduce the possibility of damage. Especially, at high speeds, the inertia effects of rotation parts must be consistently represented to enhance the stability of rotor dynamic systems. Therefore, an accurate method to model and investigate comprehensive features of a rotor dynamic system is an urgent necessary.

The most important characteristic of a dynamic system is the natural frequency. Correspondingly, another important result is the critical speed of rotation which is the most catastrophic speed. To avoid large or unpredictable failure, it is better to constrain the critical speeds as far as possible the operating speeds. This is the aim of this simulation work. Previously, many optimum methods have been used to optimize the placement of critical speeds of the rotating systems, such as genetic algorithms (GAs) as stated by Choi & Yang (2000), Ritto, et al. (2011) and gradient-based methods as presented by Pugachev (2013) [1-3]. With robust and powerful searching ability, GAs is extensively used nowadays. However, the previous researches still have some shortages of approaches and still need further study. Especially, there was not any assistant of any CAE software for optimizing complex systems in those previous works. Today, bringing further advances in technology is more and more associated with applying stochastic optimization techniques during design phase to discover optimal structures based on some CAE softwares. In particular, Bai, et al. (2012) and Tenali &

Kadivendi (2014) presented some dynamic analyses in rotor bearing systems using ANSYS [4-5].

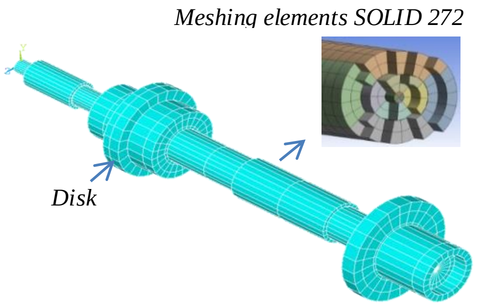
In this research, a design optimization approach including the selection of a proper physical model and a suitable finite element discretization with meaningful design variables for the stochastic solution was carried out using ANSYS. In particular, the APDL was used to model the finite system and put for a design optimization problem. Multi-objective GAs integrating with RSM were applied to find the optimum variables of the system. The optimum results show the perspective method used to optimize complex systems quickly.

2. MODEL AND ANALYSIS APPROACH

2.1 Model

For the below numerical simulation of the optimization problem, the ANSYS finite element program was applied to find critical speeds based on Campbell diagram. The modeling procedure of the rotor bearing system is illustrated under three sections including modeling of 3D finite Timoshenko rotor as BEAM 188 elements, modeling of disk mass as MASS 21 elements and modeling of isotropic bearings as COMBI14 elements in ANSYS (Fig. 1). Besides, the effects of rotary inertia unbalance forces, gyroscopic effects were taken into account. All characteristics of those elements are described in the ANSYS help manual [6].

Fig. 1 The finite element model of the Nelson rotor [7]



To prepare for optimization problem, the APDL was conducted to create the input variables and output results. There are a total of 1,296 elements and 19 center nodes after meshing the model. Some of the known parameters, such as the material properties, the bearing stiffness, are shown in Table 1.

Table 1 Calculation parameters

Parameters	Value
Young's modulus	207GPa
Poisson's ratio	0.3
Density	7806 kg/m^3
Bearing stiffness k	4.378 × 10^7 N/m
Disk mass	1.4 kg
Disk polar inertia	0.0022 kg.m^2
Disk diametral inertia	0.00136 kg.m^2
Unbalance force	85 ×10^{-3} (kg.m)

2.2 Analysis approach

Recently, the transfer matrix method and FEM often adopted to deal with rotor dynamic system. As the computer technology develops, the FEM become very popular to analyze a rotor dynamic system. The dynamic behavior of a rotor system is described by the discrete differential equation in the stationary frame of reference [5].

$$[M]\{\ddot{u}\} + ([C] + [C_{gyr}])\{\dot{u}\} + [K]\{u\} = \{F\} \qquad (1)$$

where $[M]$ is mass matrix, $[K]$ is stiffness matrix, $[C]$ is damping matrix, and $[Cgyr]$ is gyroscopic matrix, $\{F\}$ is force vector, $\{u\}$ is generalized coordinate vector. Shape functions were used as in Ref. [7].

The behavior is expressed by complex eigenvalues and eigenvectors.

$$\{u_i\} = \{\sigma_i \pm \gamma_i j\} e^{-(\sigma_i \pm \omega_i j)t} \qquad (2)$$

In the complex conjugate pair of eigenvalues $\alpha = \sigma_j \pm \omega_j$, the real parts σ_j represent the stability of the system and the imaginary parts ω_j frequencies represent the steady-state circular frequency of the system. With gyroscopic effects, ω_j are the functions of the rotational speed. These functions are plotted as Campbell diagram with forward (FW) and backward (BW) whirl curves as given in Fig. 2. Critical speeds are determined as intersection points between frequency curves and excitation line. With synchronous harmonic, excitation order n is equal to one.

To solve the optimization problem, the GAs proposed by Goldberg (1989) is utilized. With stochastic rules, the GAs has been extensively used to find global optimum variables for complicate, convex and noisy objective functions, especially multi-objective GAs by Deb (2001) [8]. Therefore, this optimum method was used in this paper. As usual, the procedure of a GAs to solve an optimum problem is shown as follows

START →Create initial population → Selection and reproduction → Crossover → Mutation → Evaluation of Fitness → Run until stopping criteria → Create new population → Report the best individual.

The DOE presented by Montgomery (1984) is one of the best methods to closely connect a subject to an engineering optimization problem [9]. It can be used in the post-optimal analysis phase. Additionally, the method combines DOE and robustness of a system is RSM as stated by Venter (1998) [10]. In these methods, an initial experiment is set up and conducted. The objective function is then approximated by usually a second order polynomial response surface. The experiment can be repeated so that a more accurate response surface is obtained.

3. NUMERICAL EXAMPLE AND RESULTS

3.1 Modal analysis

The modal analysis was implemented to find the critical speeds of the system based on the given parameters in the Table 1 and Ref. [7]. In the Fig. 2, the Campbell diagram shows the variation of frequencies with respect to spin velocities. With synchronous harmonic or only symmetric unbalance, the excitation order is equal to one. The received critical speeds were given in Table 2. Moreover, to see clearly the behaviors of the system, the mode orbits of each center node at the natural speeds were provided in Fig. 3.

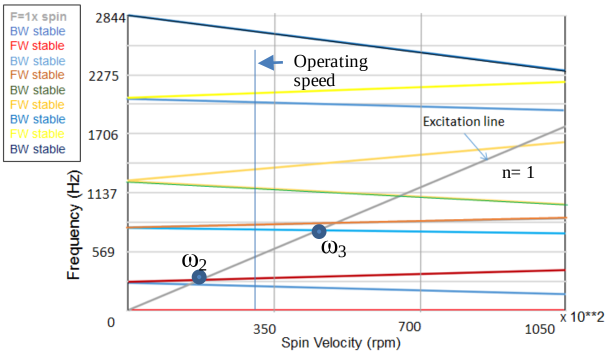


Fig. 2 The Campbell diagram of the rotor

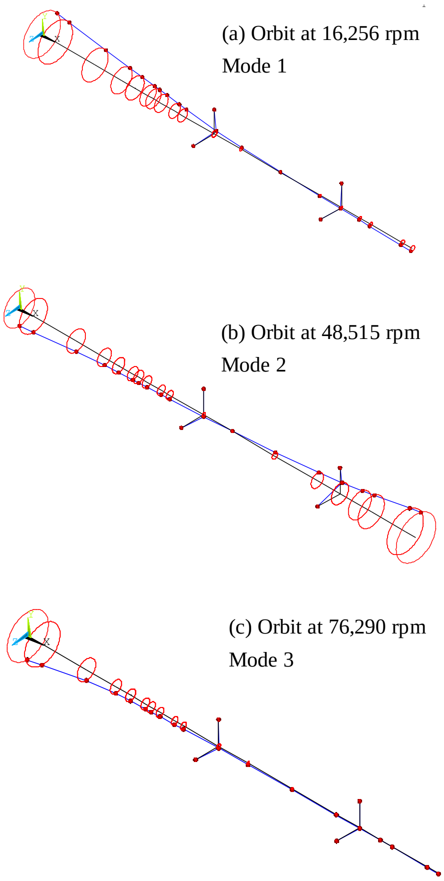


Fig. 3 Mode orbits of the rotor at the natural speeds

3.2 Design optimization problem

In general, an optimization is used to find a set of design parameters, $x=(x_1, x_2, \ldots, x_n)$, that can in some way be defined as optimal. The objective functions f(x) were minimized or maximized subject to constraints in form of equality $g_{eq(x)}$, inequality $g_{ieq(x)}$ or nonlinear constraints $c_{(x)}$ and parameter lower and upper bounds, x_{lb}, x_{ub}. In this paper, all these forms were given below.

The multi-objective functions

Minimize: $f_{1(x)} = \omega_2$ and $f_{2(x)} = -\omega_3$

Subject to $\omega_2 \leq 17{,}128$ (rpm)

$46{,}700 \leq \omega_3$ (rpm)

The variables and boundary:

$$0.045 \leq d_6 \leq 0.07 \text{ (m)}$$

$$0.02 \leq d_{10} \leq 0.03 \text{ (m)}$$

$$4.378 \times 10^6 \leq k \leq 4.378 \times 10^8 \text{ (kN/m)}$$

where ω_2, ω_3 are the second and third critical speeds near the operating range; d_6, d_{10} are the disk diameter located at the station 3 and the rotor diameters at the stations 4 and 6, respectively [7].

For DOE, the Latin Hypercube Sampling Design was used to find the design points versus the given variables with 100 samples. Applying RSM with Kriging response surface type, the response points were received. It is seen that the variable d_6, the disk diameter, has the greatest influence on the dynamic characteristics of the system in Fig. 4. These results are very necessary for engineers in design stage of a rotor system.

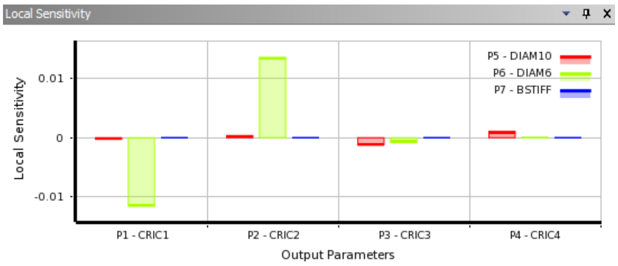


Fig.4. The local sensitivity of variables

The parameters of the GAs for this paper are chosen as follows: number of initial samples =100; number of samples per iteration =100; crossover probability = 0.7. After 1711 iterations, the optimum point and optimum results were found as given in Tables 2-3. Figures 5 and 6 show the unbalanced response at the disk point corresponding to the original and optimum design. It is evident to see that the response of the optimum design decreased in the operating range.

Table 2 Variables of original and optimum case

Name	Original variable	Optimum variable
d_6	0.066 (m)	0.0451 (m)
d_{10}	0.0254 (m)	0.02 (m)
k	4.378×10^7 (N/m)	1.2272×10^8 (N/m)

Table 3 Critical speeds of original and optimum case

Name	Whirl	Reference results [7]	Original speed (rpm)	Optimum speed (rpm)
ω_1	BW	15,470	15,478	13,223
ω_2	FW	17,159	17,128	15550
ω_3	BW	46,612	46,711	60,790
ω_4	FW	49,983	50,095	70,602
ω_5	BW	64,752	64,875	91,085
ω_6	FW	96,457	95,636	None

CONCLUSION

The goal of optimization problem in the rotor dynamic system using multi-objective GAs based on

ANSYS was accomplished. The Campbell diagram was plotted to find the natural frequencies and the critical speeds. The optimum results show that the disk diameter has the largest effect on the dynamic characteristic of the system. Furthermore, the optimum point was found in order to reduce the unbalanced response of the system. Additionally, it could be concluded that the ANSYS program offers comprehensive capabilities and enables solution of large-scale, complex models and could be an effective tool for rotor dynamic analysis in many aspects.

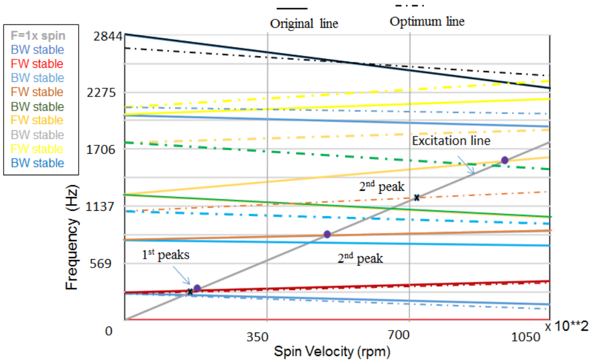


Fig. 5 Campbell diagram between original and optimum rotor

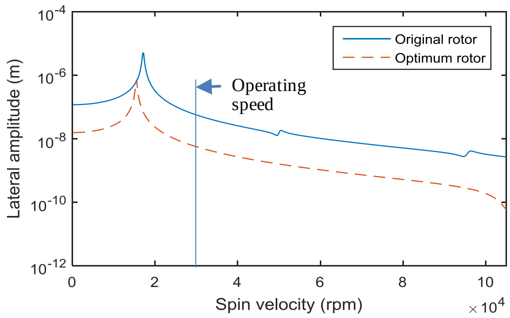


Fig. 6 Unbalance response between original and optimum rotor

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